Optical Synthesis Imaging

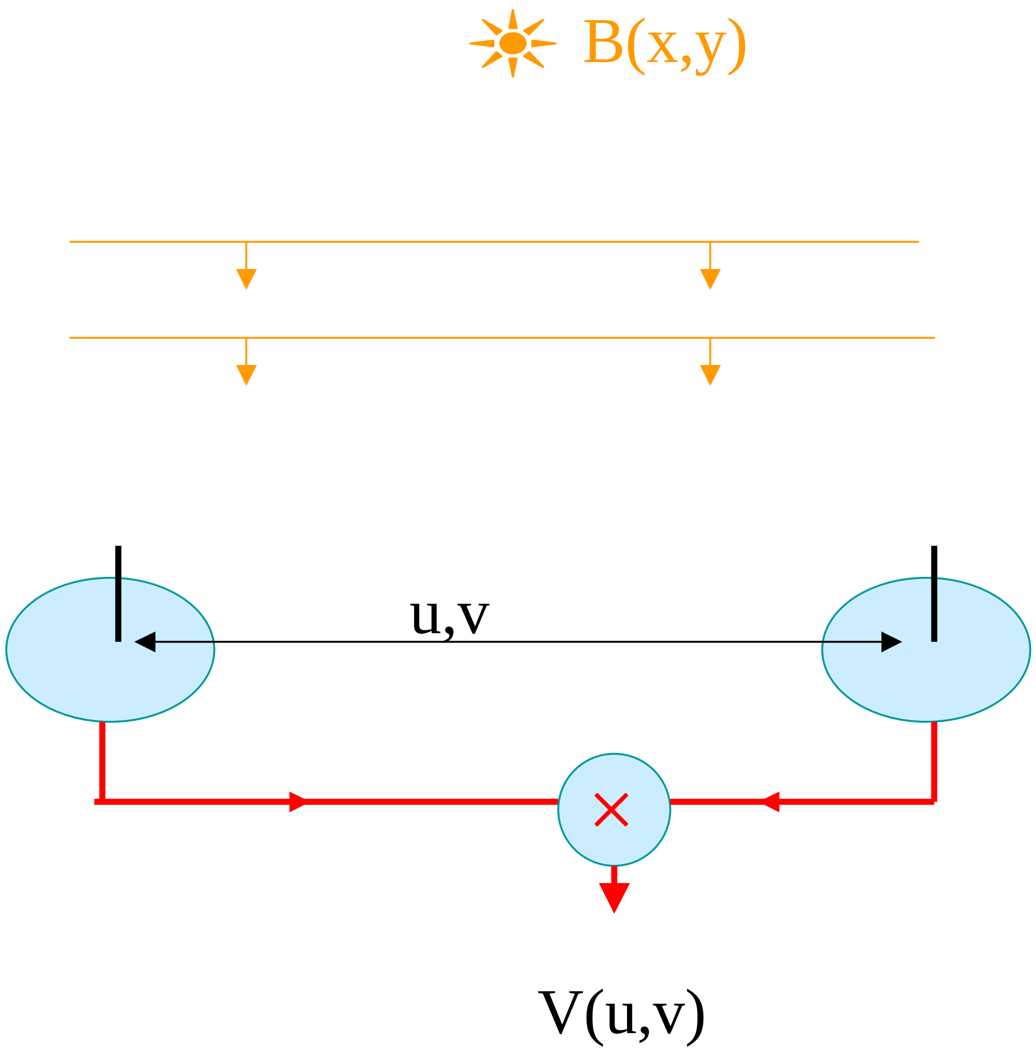
David Buscher
Michelson Summer School
Flagstaff

Synopsis

- Why imaging?
- Simple image reconstruction.
- The importance of phase information.
- Closure phase.
- Measuring the closure phase.
- Visibility calibration.
- Nonlinear image reconstruction.
- Example from real data - Betelgeuse.

A simple interferometer

- Two-element radio telescope, no atmospheric perturbations.
- Measures V(u,v)=F.T.{B(x,y)}
- Can move the elements to sample the u,v plane as we like.

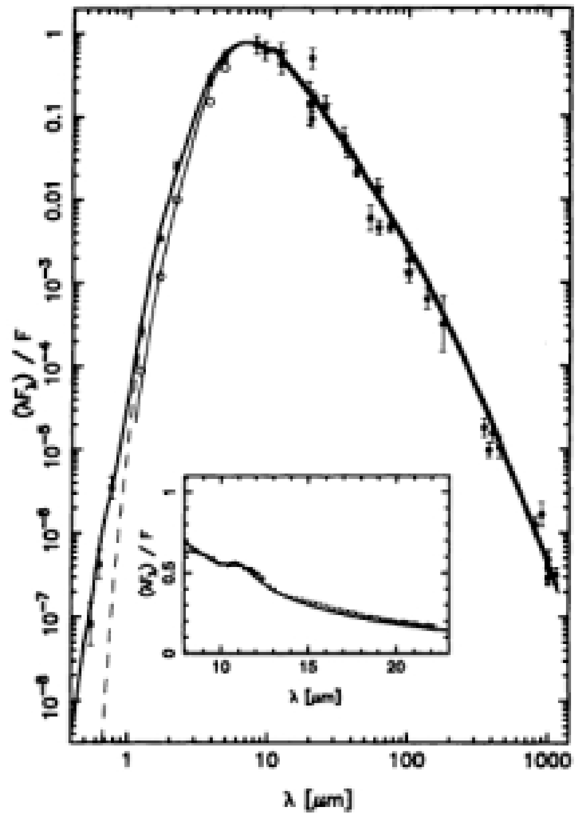


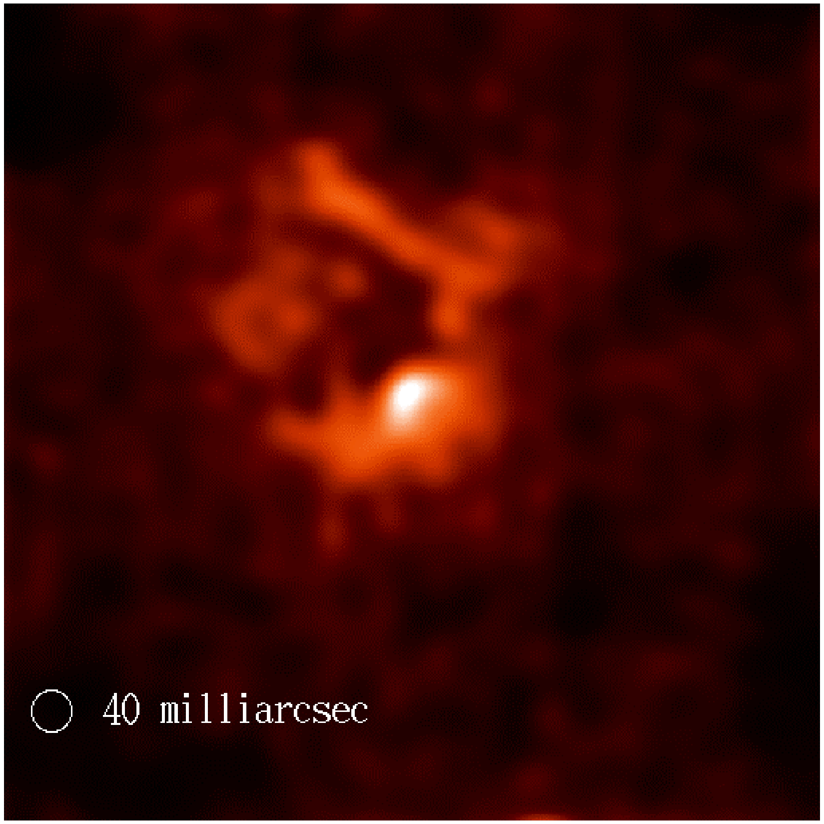
Why imaging?

- If we measure V(u,v) for all u,v, we can use an inverse Fourier transform to get an image of the source.
- An alternative is to measure V (or |V|) at a small subset of u-v points and then fit an astrophysical model with a small number of parameters - **modelfitting** (sometimes called “parametric imaging”).
- This can be dangerous.

Modelfitting vs imaging

SED of IRC+10216: spherically symmetric model (Ivezic & Elitzur, 1996)





Actual distribution of 2 micron flux (Tuthill et al, 2000)

Imaging

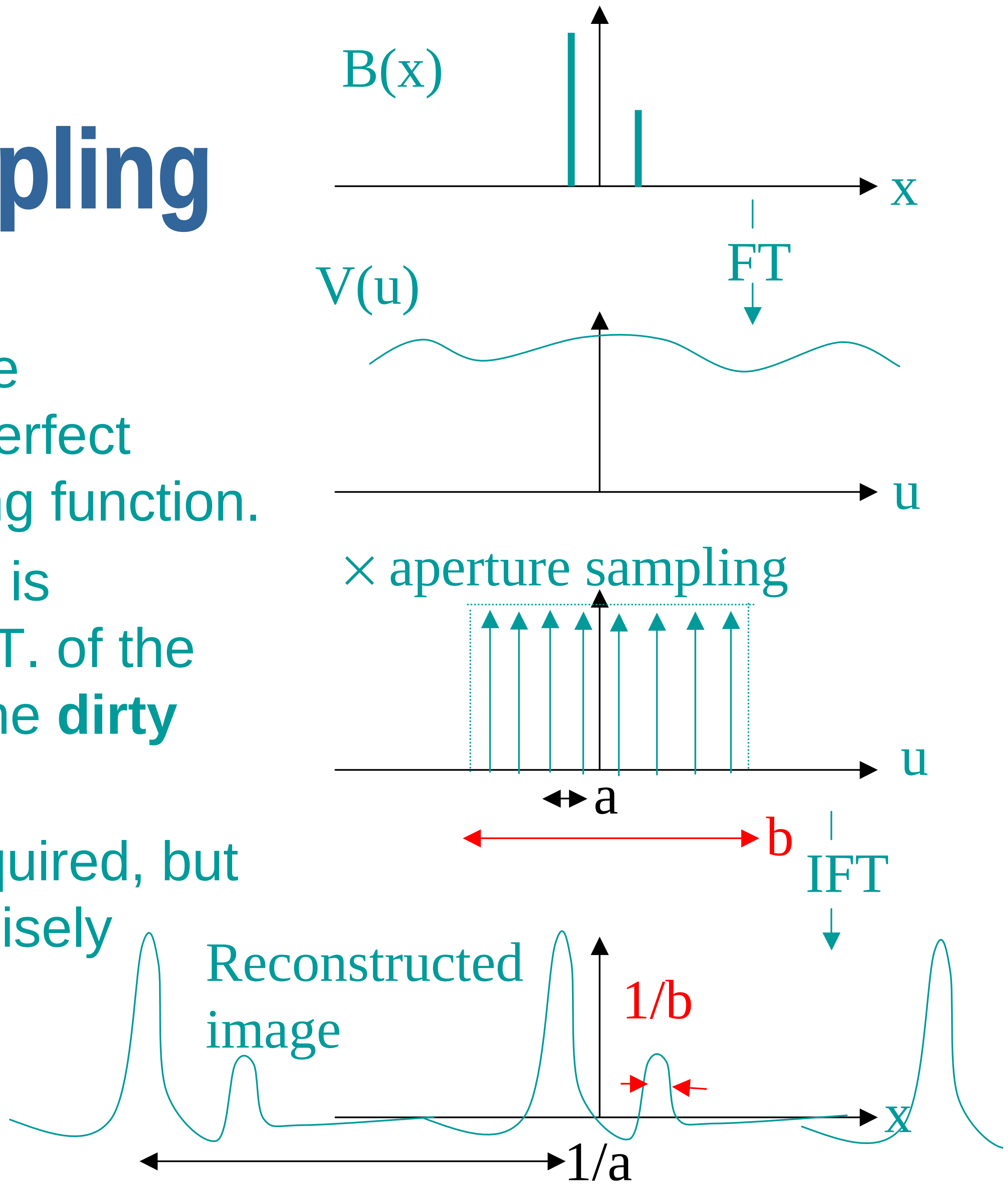
- Moral: there is no substitute for model-independent images.
- This conclusion will lead us down a tortuous path:
  - U-V coverage.
  - Closure phase.
  - Visibility calibration.
  - Nonlinear image reconstruction.

U-V coverage

- Can only sample a discrete set of points in the U-V plane – call this sample the **synthetic aperture**
  - The aperture is finite.
  - The aperture is **dilute**.
- Can tackle both of these using the convolution theorem.

Aperture sampling

- Effectively multiply the measurements of a perfect aperture by a sampling function.
- Reconstructed image is convolved with the F.T. of the sampling function – the **dirty beam** or **PSF.**
- **Deconvolution** is required, but the dirty beam is precisely known.



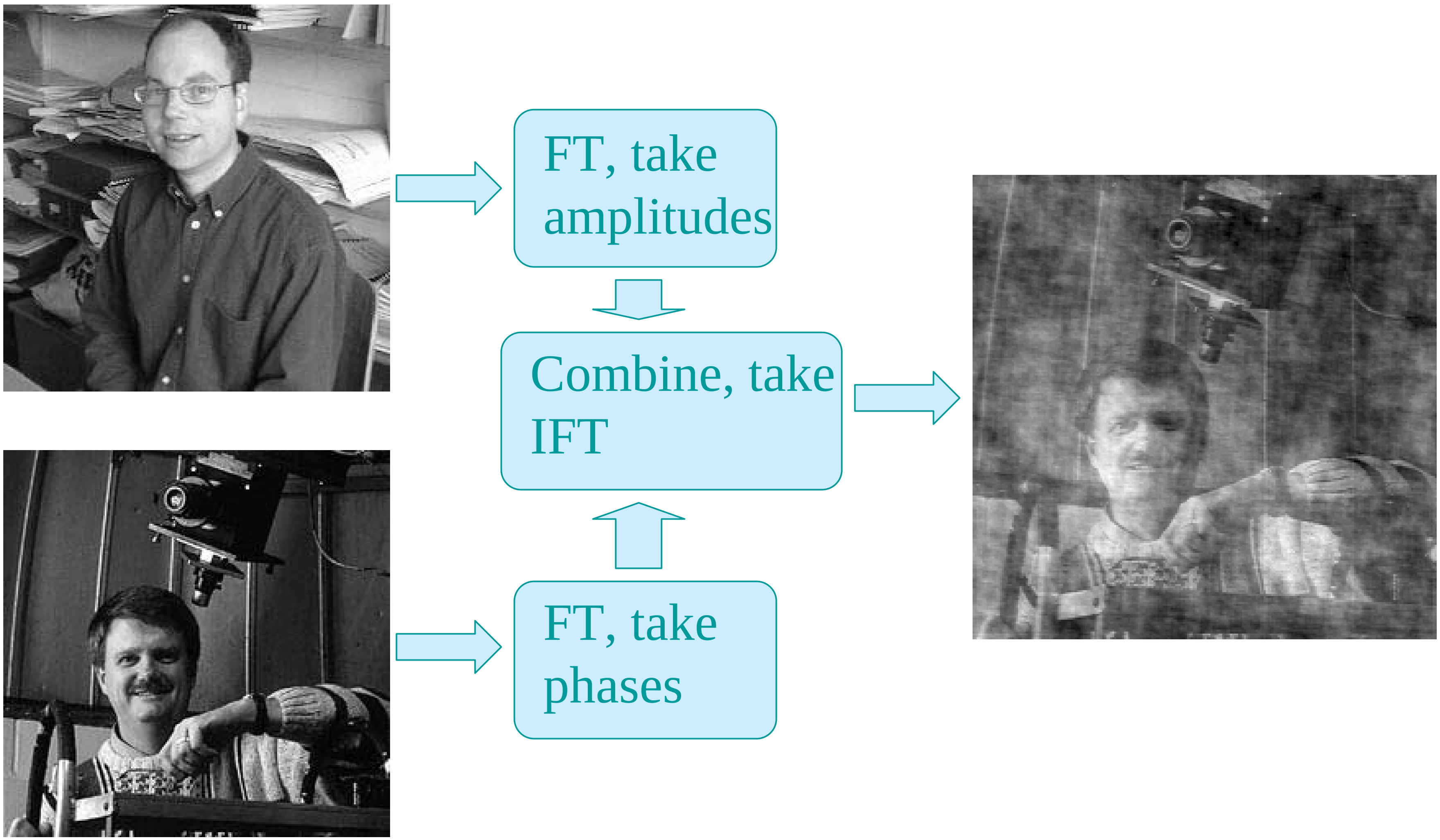
Choosing a U-V coverage

- Strongest constraints are practical:
  - Amount of time to reconfigure telescopes;
  - Earth rotation;
  - Local topography;
  - Bootstrapping.
- The convolution theorem is again useful:
  - If the source is known to be a finite size, this is the same as an infinite source truncated with a tophat of size θ_{max}.
  - Hence V(u,v) is correlated on scales of $\lambda/\theta_{max.}$
  - No point sampling on scales much finer than this.

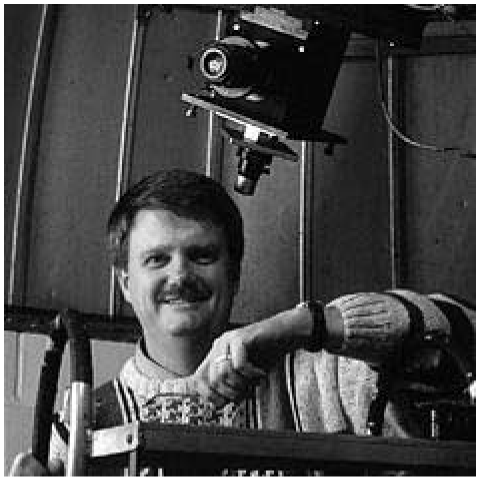
The phase problem.

- Now we add the atmosphere (in a simple form).
- Adds a random phase (rms >> 2π) over each aperture.
- This means that only $|V(u,v)|$ is easily measured – phase information is “lost”.
- In principle, you can reconstruct images from Fourier modulus information alone.
- In practice, this works only with perfect data.

Why you need phases



Why you need phases



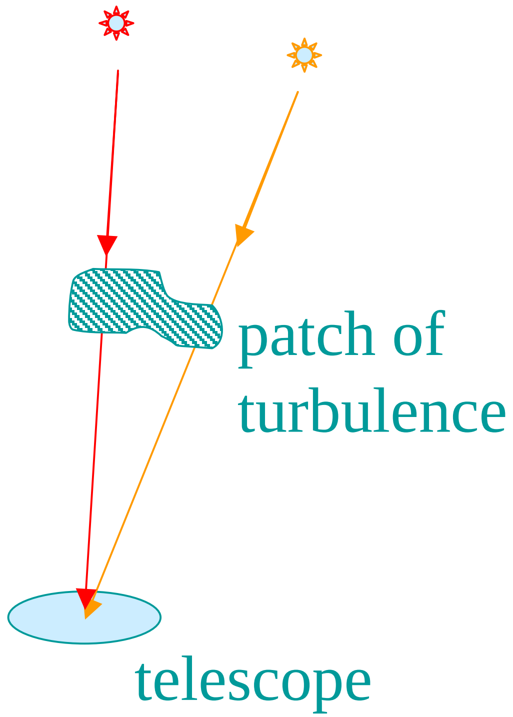
FT, take amplitudes

Combine, take IFT

FT, take phases

Methods of getting the phase (i)

- From an external phase reference:
  - Nearby guide star:
    - internal metrology
    - limited sky coverage
    - anisoplanatism.
  - Laser reference – possible (balloons?) but challenging



Methods of getting the phase (ii)

- Self-referenced methods – use the source itself.
  - Phase referenced to a different wavelength
    - Source-dependent
    - Need to know where group delay centre is
    - Need to know atmospheric path & dispersion
      - Water-vapour variations can be important
  - Phase referenced to other baselines
    - Closure phase

The closure phase (i)

- Consider an array of N telescopes:
  - Can measure N(N-1)/2 baseline phases.
  - Subject to N-1 unknown phase perturbations.
  - Can therefore solve for (N-1)(N-2)/2 quantities which are dependent <u>only on the source phase</u>.
  - The simplest (but not the only) parameterisation of these source-dependent quantities are the **closure phases**: combinations of phases on <u>closing</u> triples of baselines.

The closure phase (ii)

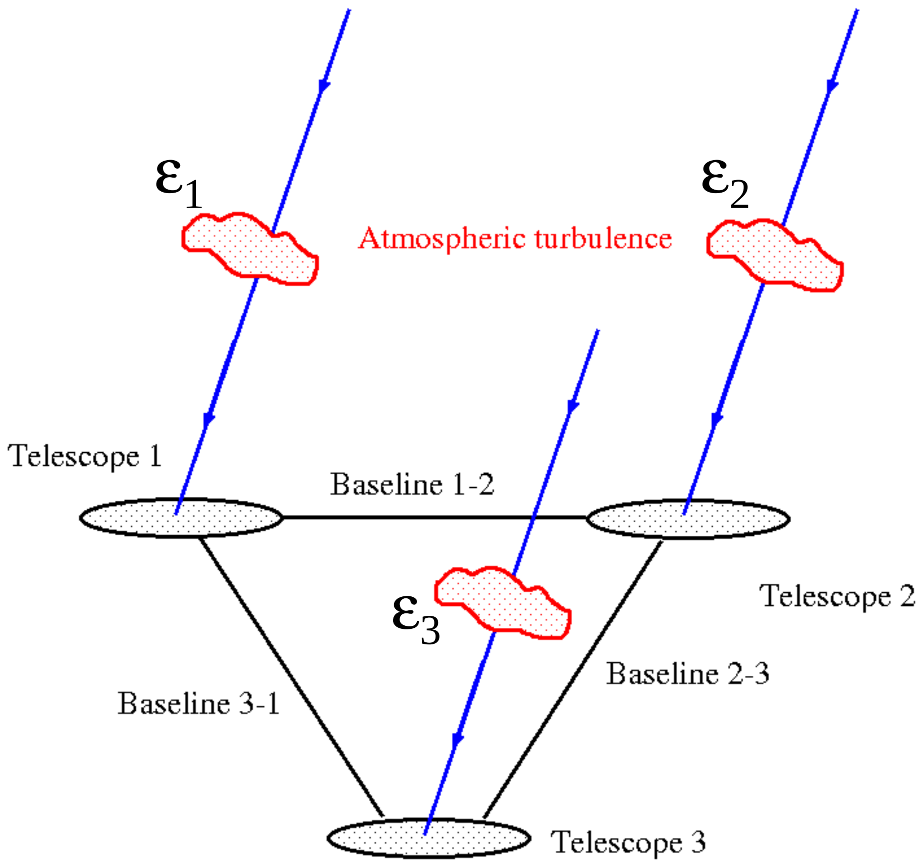
Measured Source "Antenna"

$$\Phi_{12} = \phi_{12} + \varepsilon_1 - \varepsilon_2$$
$$\Phi_{23} = \phi_{23} + \varepsilon_2 - \varepsilon_3$$
$$\Phi_{31} = \phi_{31} + \varepsilon_3 - \varepsilon_1$$

Combine $\Rightarrow$

$$\Phi_{12}+\Phi_{23}+\Phi_{31} = \phi_{12} + \phi_{23} + \phi_{31}$$

- Source terms are baseline-dependent.
- Error terms are antenna-dependent.



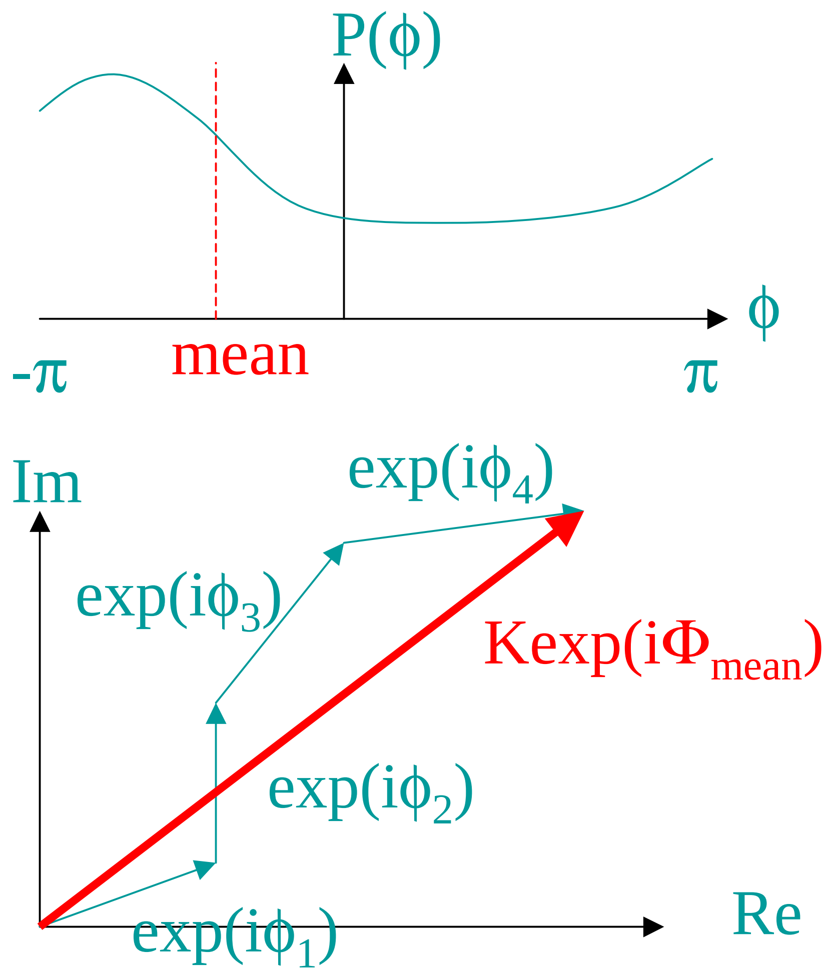
The closure phase (iii)

- The full set of closure phases is overdetermined for N > 3:
  - (N-1)(N-2)/2 independent source quantities
    - 10 for N=6
  - N(N-1)(N-2)/6 triples of antennas
    - 20 for N=6
- Higher order “closure phases” exist, e.g.

  $\Phi_{12}+\Phi_{23}+\Phi_{34}+\Phi_{41}$
  - Also immune to antenna errors
  - Worse SNR than triple

Measuring (closure) phases in noisy conditions

- Averaging phases directly leads to biases, especially in noisy conditions.
- Use the vector average to avoid bias under all noise conditions
  - Converges even when SNR<<1



The triple product

- In averaging the closure phase, can weight the vectors with the product of the amplitudes:

  phasor = $|V_{12}|\,|V_{23}|\,|V_{31}|\exp(i\Phi_{12} + i\Phi_{23} + i\Phi_{31})$

- But this is simply the product of the complex visibilities

  $$T_{123} = V_{12}V_{23}V_{31}$$

- We call this the **triple product** or “bispectrum”
  - Better SNR than unit-weighted vectors.
  - Other nice properties.

Noise on the triple product

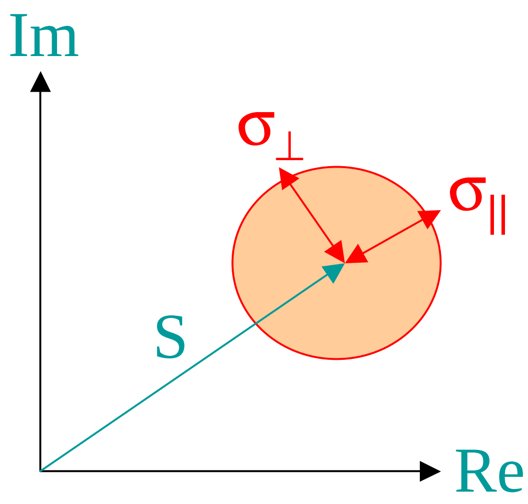
- Definition of phase error:

  $\sigma_\theta = \sigma_\perp/|S|$
- For circularly symmetric noise $\sigma_\perp = \sigma_{||}$

  $\sigma_\theta = 1/(\sqrt{2}\times SNR)$
- For SNR>> 1

  $\sigma_\theta^2(T_{123}) \cong \sigma_\theta^2(V_{12}) + \sigma_\theta^2(V_{23}) + \sigma_\theta^2(V_{31})$
- For SNR<< 1

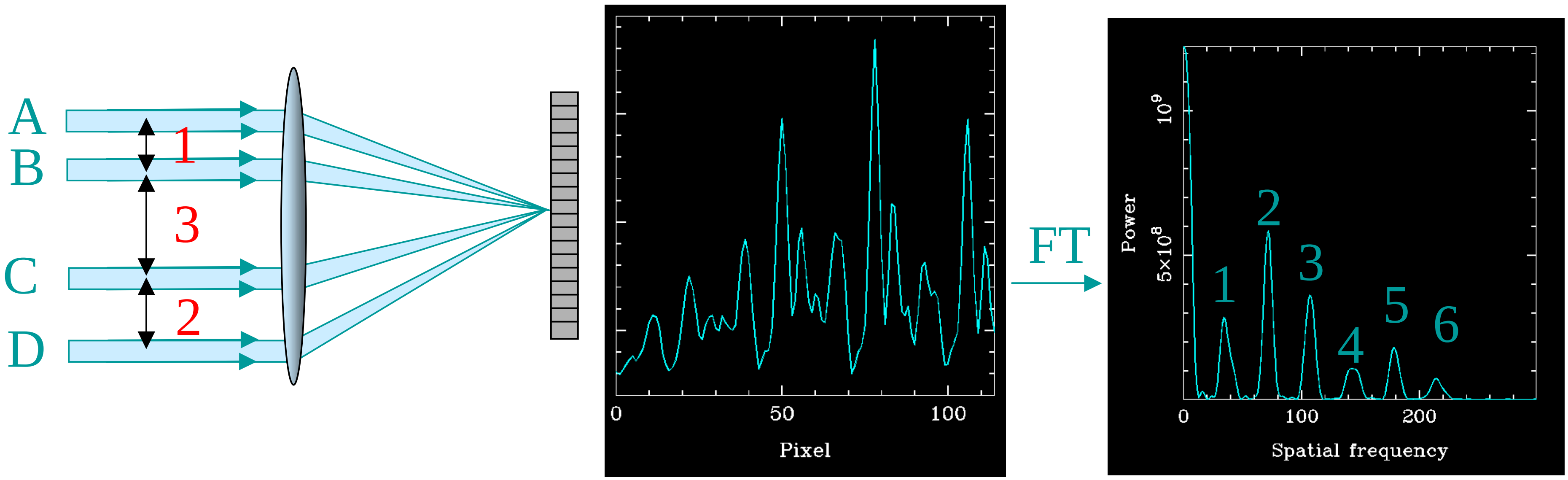
  $\sigma_\theta^2(T_{123}) \cong \sigma_\theta^2(V_{12})\sigma_\theta^2(V_{23})\sigma_\theta^2(V_{31})$
- C.f. noise on visibility modulus $\sigma^2(|V|^2) \cong [\sigma^2(V)]^2$
- However, many useful cases where two baselines have high SNR and one has low SNR
  - Low-SNR baseline “phased up” using high-SNR baselines.



Noise correlations

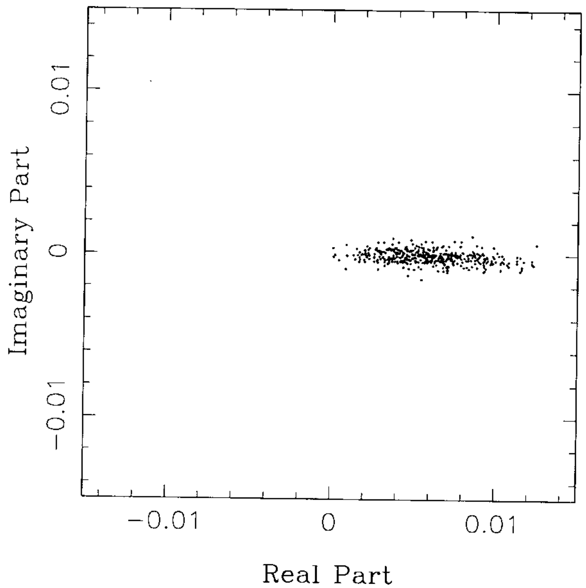
- In the high-SNR regime, the noise on triple products sharing a common baseline is correlated.
- In the low-SNR regime, the noise on <u>all</u> triple products is uncorrelated.
  - Means that measuring the full set of closure phases helps to beat down the noise.
- Radio VLBI corresponds to the high-SNR regime.
- Optical interferometry usually corresponds to the low-SNR regime – can take 1000's of measurements to get low-error averaged data.
  - Radio imaging programmes don't make use of all the information in optical datasets.

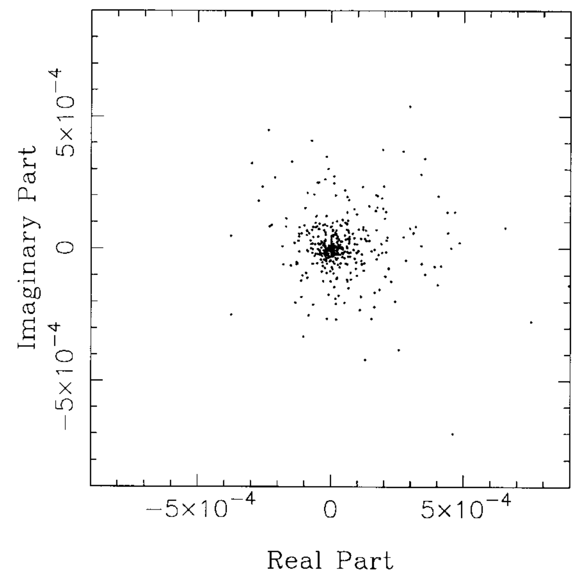
Measuring closure phases in practice – image plane



- Take many fast exposures on detector.
- Choose a triple of <u>apertures</u>, e.g. A, B, C and get a corresponding triple of spatial frequencies 1,3,4 (1,2,3 will <u>not</u> work!).
- Multiply the complex Fourier amplitudes $T_{ABC}=V_1\,V_3\,V_4$.

Measuring closure phases (cont'd)



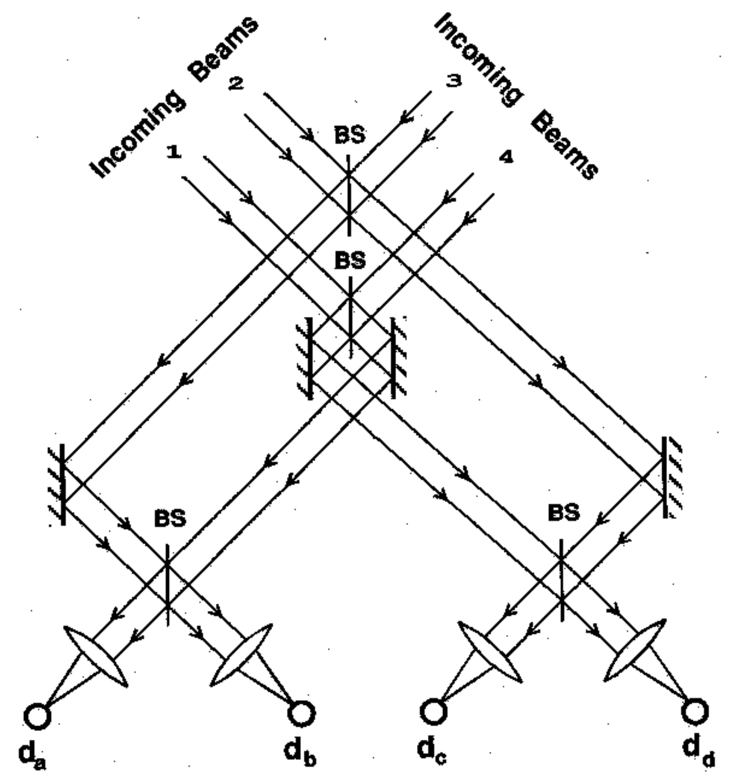


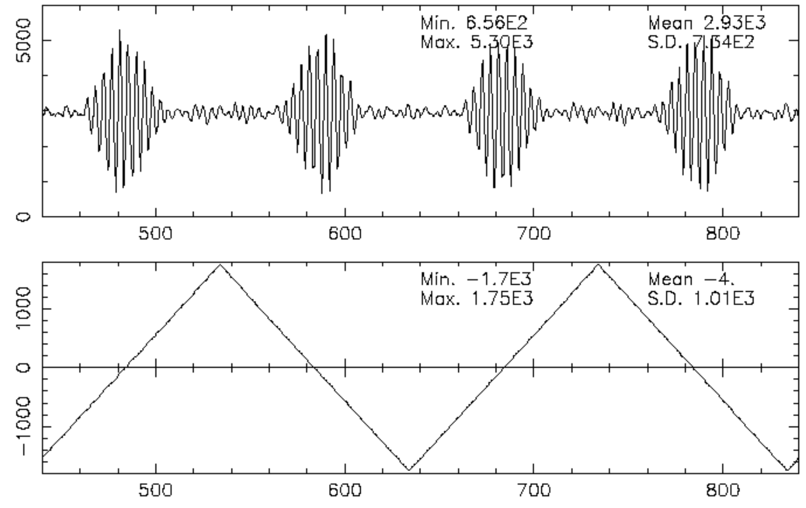
- Average over many samples and take argument – closure phase.
- Important to get a closing set of spatial frequencies

$$u_1+u_3+u_4 = 0.$$

Pupil plane combination

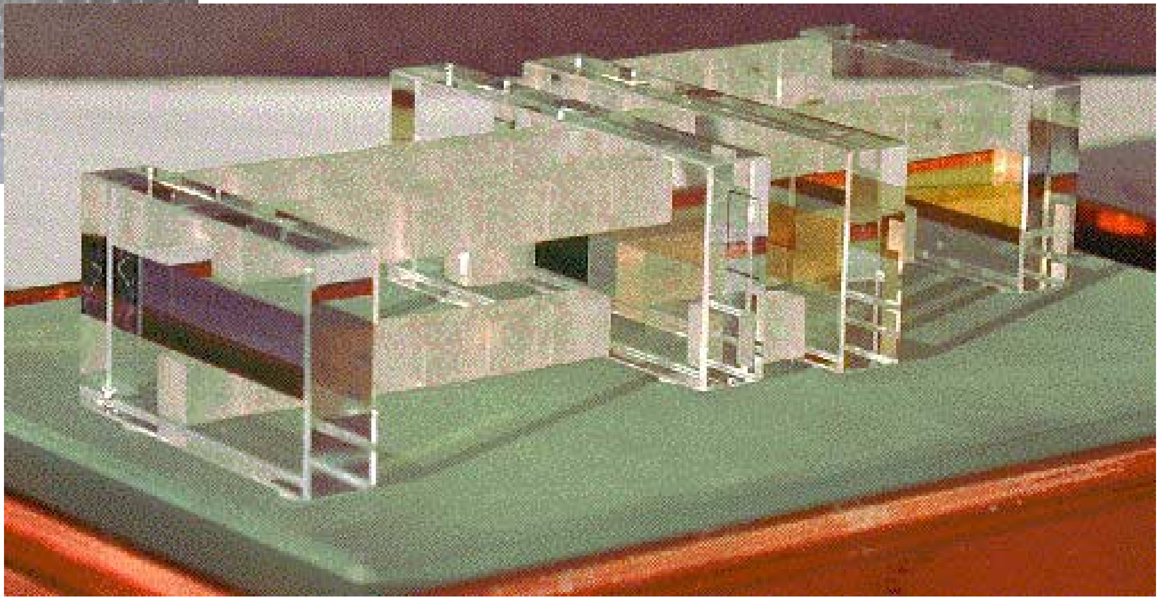
- Interference occurs on beamsplitters.
- Aligned to give a single fringe across the beam.
- Focus onto single-pixel detectors, e.g. APDs.
- Fringe signal detected by temporal path modulation.





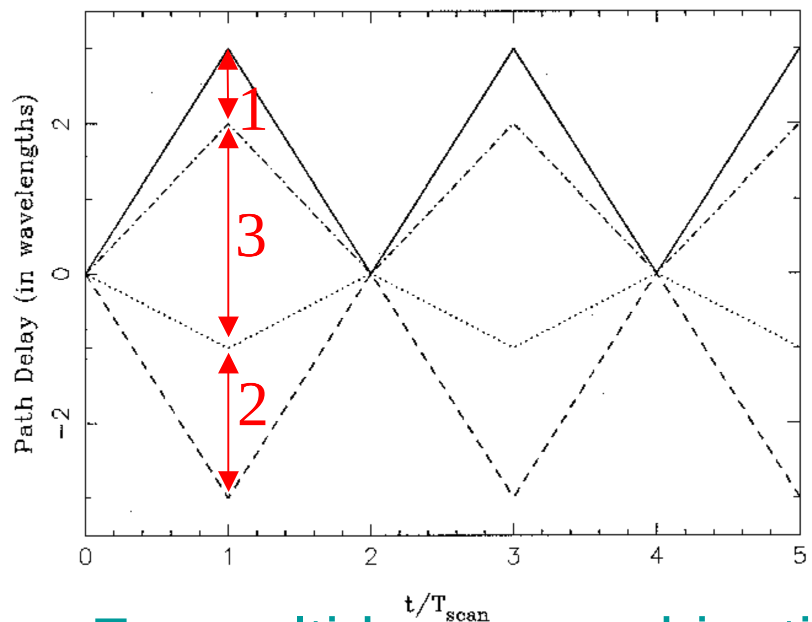
Pupil plane combination (cont'd)

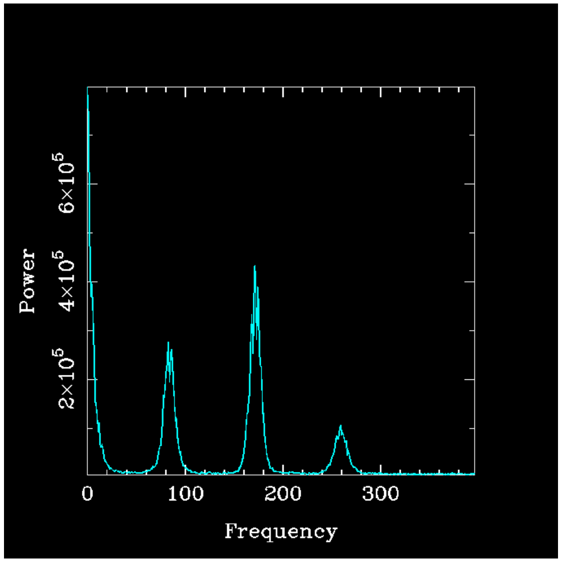
Optical table in
COAST bunker



New miniature
beam combiner

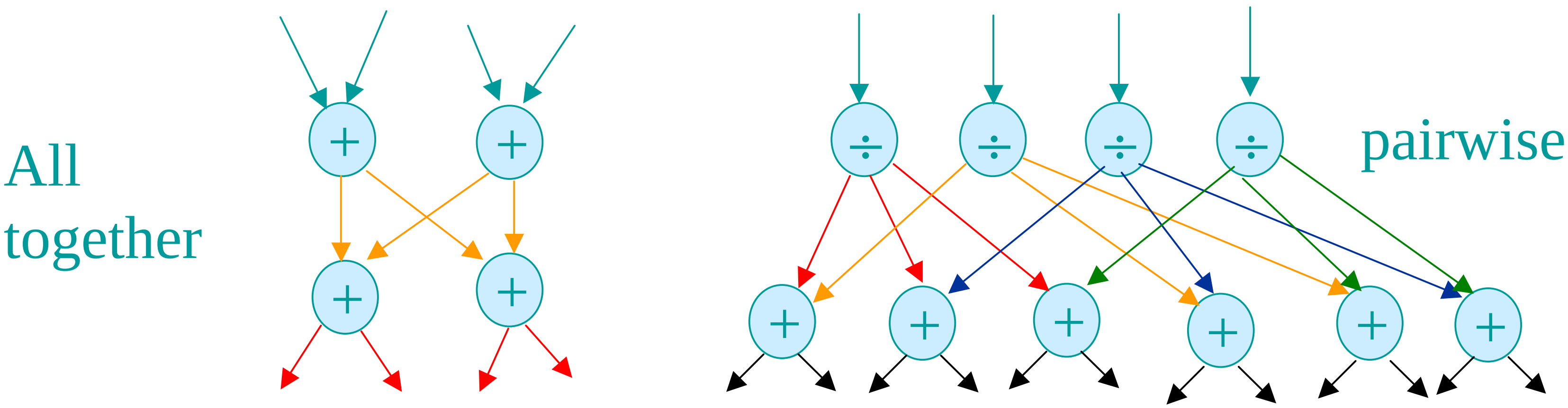
Pupil plane combination (cont'd)





- For multi-beam combination need to have fringes from different baselines at different frequencies.
- Corresponds to different modulation <u>speeds</u> $d\varphi/dt$ for different beams.
- Temporal F.T. $\rightarrow$ complex visibility phasors, then same as image plane combination.

Pairwise vs all together

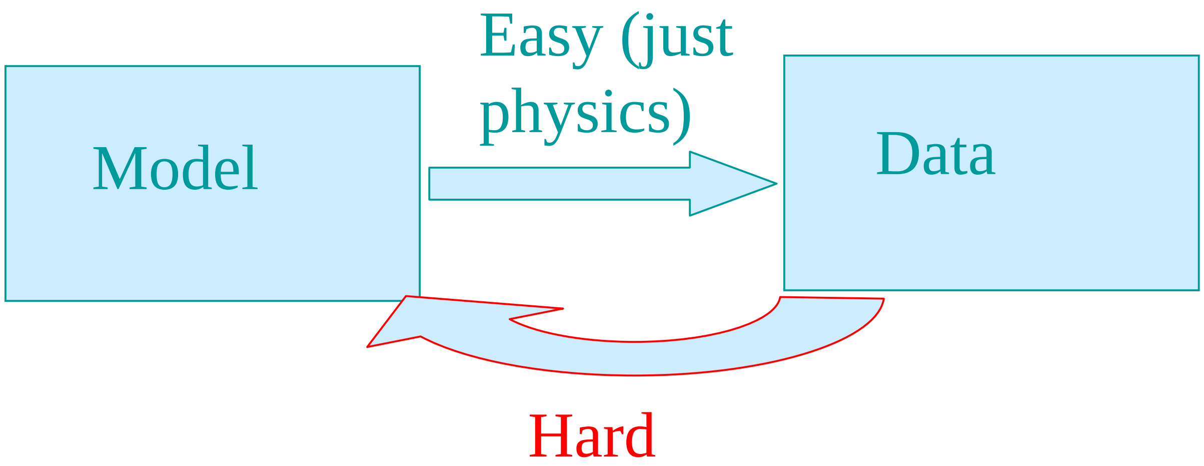


- SNR is comparable.
- All together is immune to internal path errors – no closure phase calibration necessary.
- Pairwise has no amplitude crosstalk between baselines – all together requires well-separated set of frequencies.

Visibility calibration

- Have so far been considering a wavefront error which is fixed in time and flat across each telescope.
- Higher-order effects bias the visibility amplitude to smaller values.
- Can calibrate this visibility reduction by measuring the visibility on a point source.
- Atmospheric seeing varies on all timescales, so the visibility reduction is time-dependent.
  - Need to calibrate often.
- Spatial filtering using e.g. monomode fibres helps with this.

Image reconstruction



- An **inverse problem:**
  - Forward transform, e.g. from a sky brightness distribution to measured visibilities and closure phases, is easy to do.
  - Inverse transform hard to derive, may not be unique due to noise & missing data.

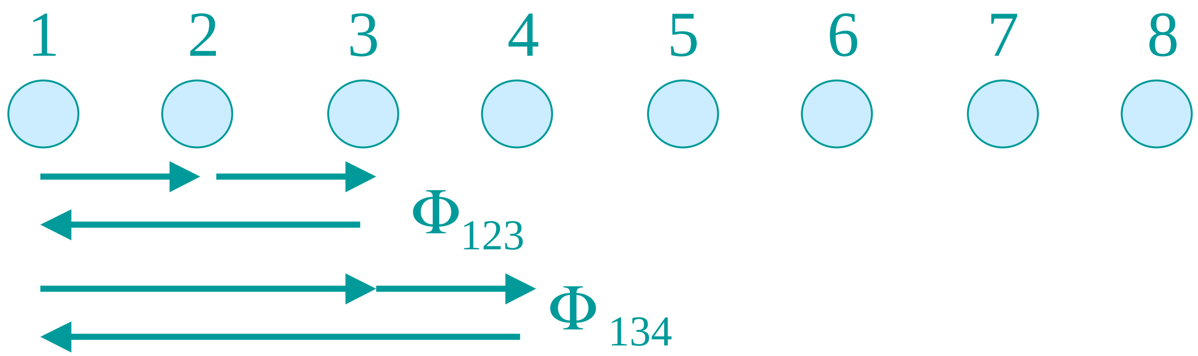
Inverse problems: Bayes' theorem

- Bayes theorem:
  - Tells us <u>quantitatively</u> the best thing to do with uncertain information.
  - prob. of model given data $\propto$ prior prob of model x prob of data given model.
- Recipe:
  - Generate all possible models (tedious but possible).
  - Find the likelihood that each model would have generated the data (easy).
  - The one which best predicted the data wins (modulo prior information).

Bayes' theorem and closure phases.

- The interpretation of a closure phase is now more clear – a closure phase is a <u>constraint</u> on the set of all possible images.
- Acts in concert with all other constraints
  - Amplitudes.
  - Source positivity.
  - Source finite extent.
- No need to invent a special procedure for converting closure phases to images – just use Bayesian recipe with the forward transform.

Recursive phase reconstruction



- <u>Not</u> Bayesian.
- Algorithm:
  - Arbitrarily choose a phase for baseline 12 (which is also that for 23, 34, …)
  - Using Φ_{123} can now derive phase on baseline 13.
  - Repeat to generate all phases.
  - Combine with Fourier amplitudes & FT $\rightarrow$ image.

Limitations of recursive phase reconstruction

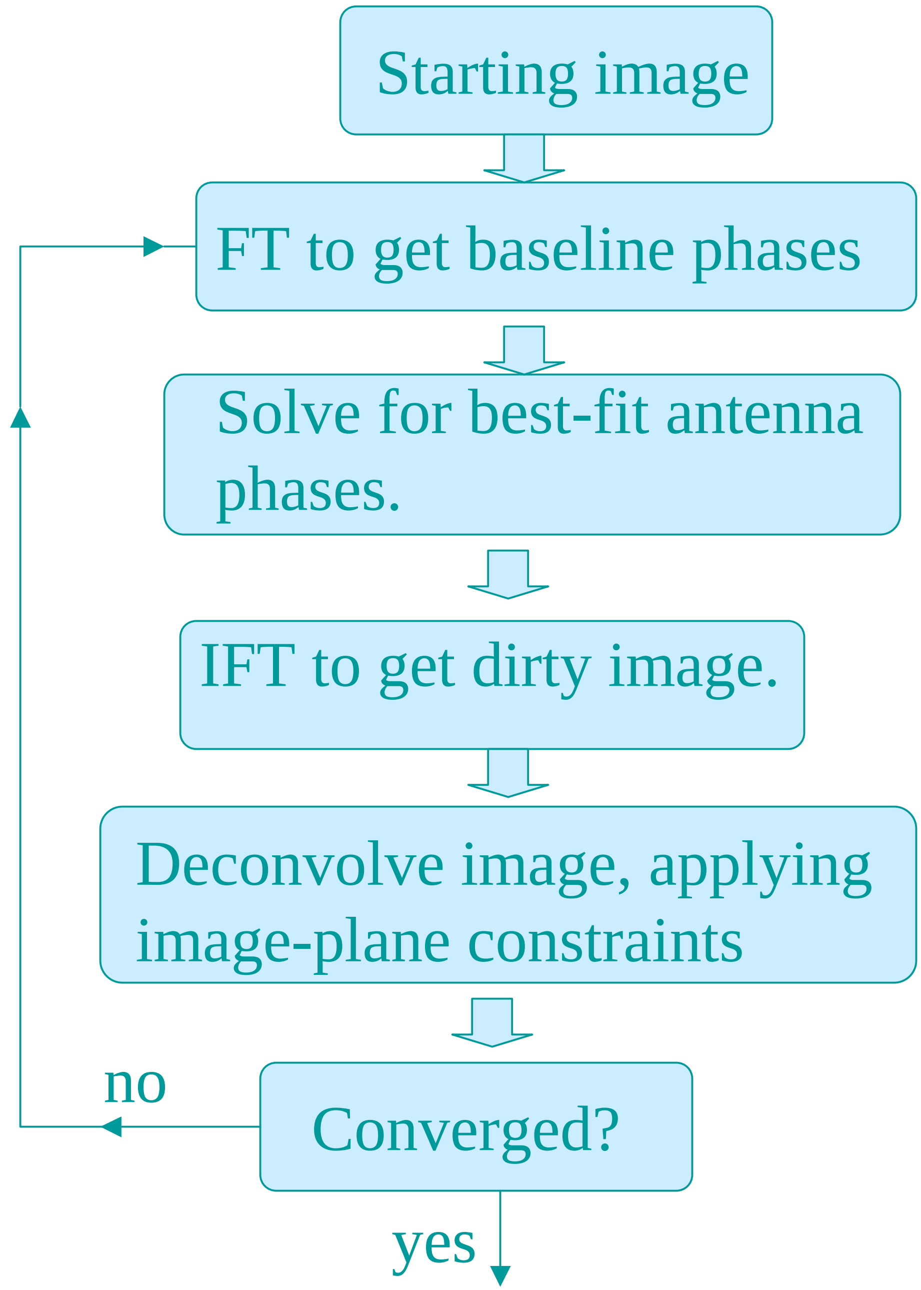
- Needs redundant array – wasteful of telescopes.
- Noise propagation is poor.
- Still need to deconvolve image.
- Doesn’t make use of image-plane constraints in derivation of phases.

However:

- It illustrates that it is by combining closure phases that we constrain phases.
- The closure phases do not constrain the phase completely – source position is unconstrained.

Self calibration

- Radio VLBI imaging method.
- Forward model explicitly includes the antenna phase errors.
- Solves for image-plane constraints and data at the same time.
- Does not depend on starting image (usually!)



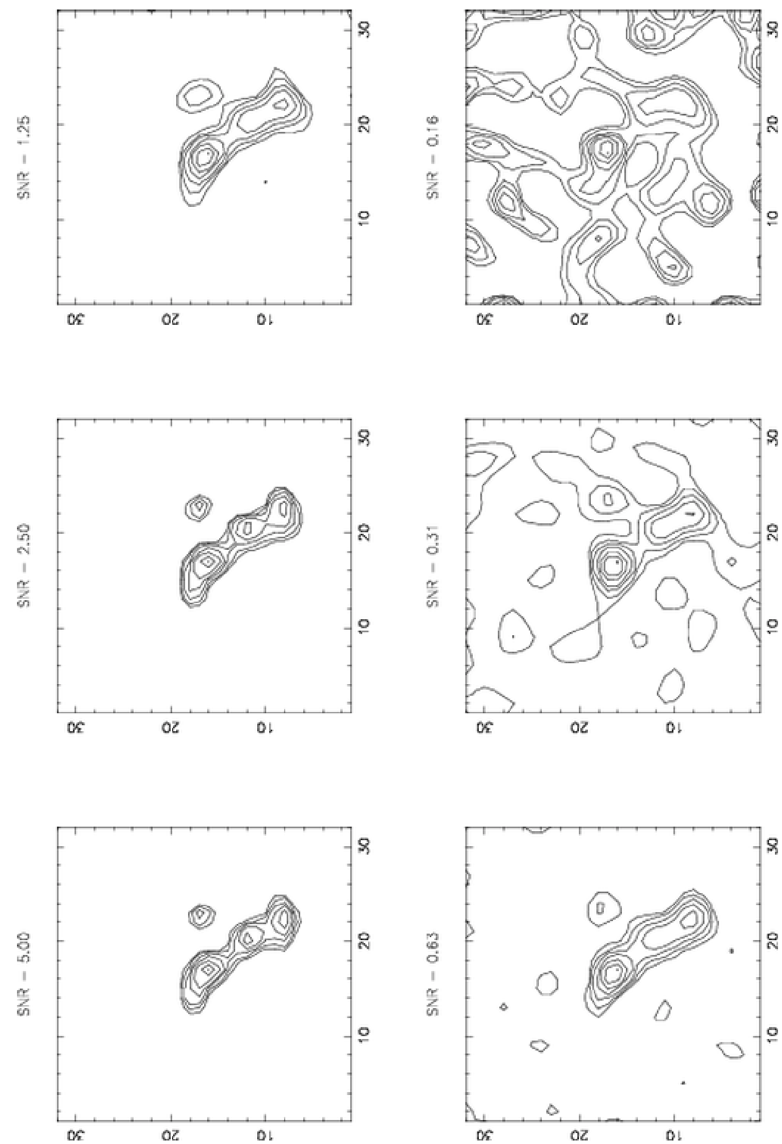
Direct reconstruction (BSMEM)

- Limitations of self-cal
  - Noise model on closure phases assumes high SNR.
  - Cannot use disjoint sets of amplitudes and closure phases.

- Alternative method: direct comparison of models and amplitudes, triple products.
  - Model is a set of pixel brightnesses.
  - Use gradient-descent methods to efficiently find best-fit image.
  - Maximum entropy used to enforce positivity.
  - All constraints applied simultaneously
    - Deconvolution & phase retrieval in one step.

BSMEM results

- Classic self-cal breaks down when the effective SNR per baseline is <~2.
- Direct reconstruction can return good results under these conditions, provided there is a large number of different closure phases.
- Effectively averaging different closure phase information together.



Imaging example: Betelgeuse

Mon. Not. R. astr. Soc. (1990) **245**, *Short Communication*, 7P–11P

Detection of a bright feature on the surface of Betelgeuse

D. F. Buscher,[1] C. A. Haniff,[1,2] J. E. Baldwin[1] and P. J. Warner[1]

[1] *Mullard Radio Astronomy Observatory, Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE*
[2] *Palomar Observatory, California Institute of Technology, Pasadena, CA 91125, USA*

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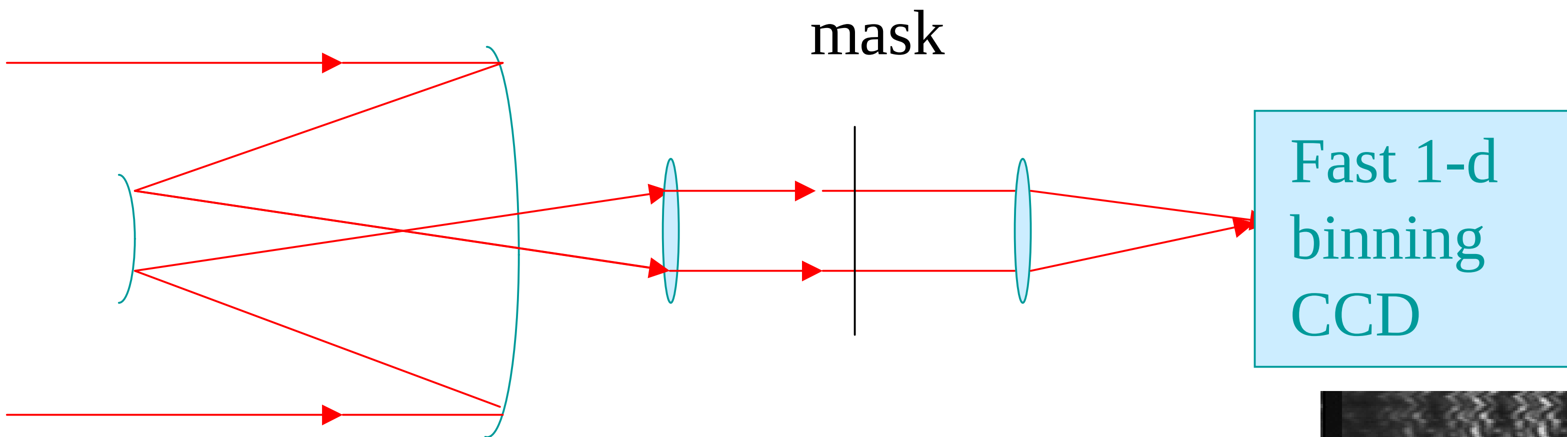
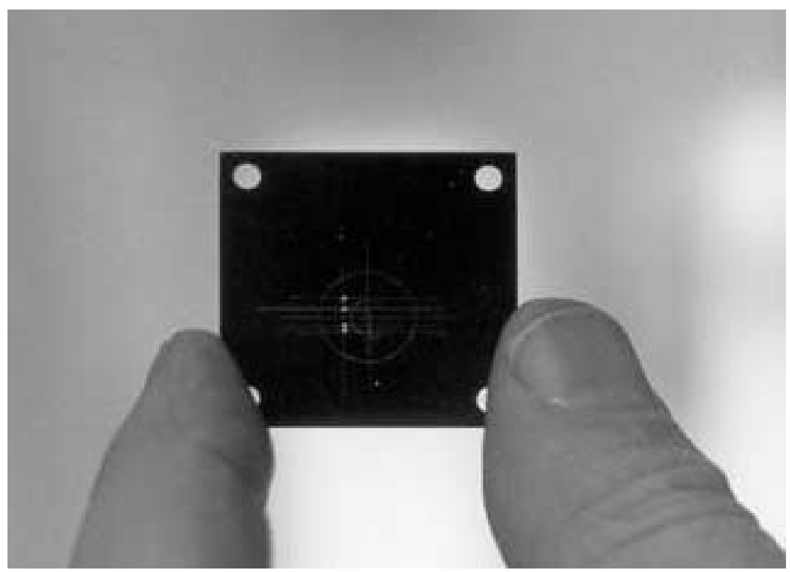
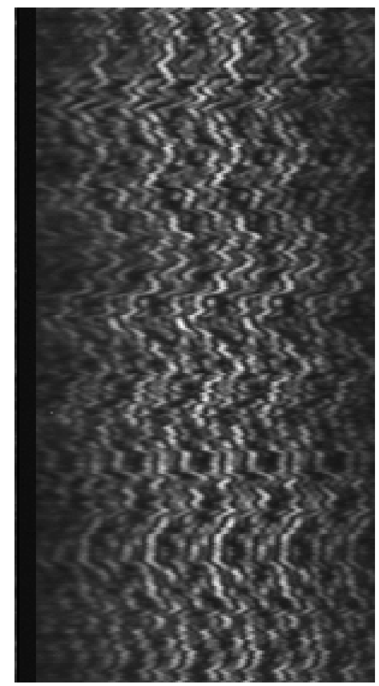
SUMMARY

We present high-resolution images of the M-supergiant Betelgeuse in 1989 February at wavelengths of 633, 700 and 710 nm, made using the non-redundant masking method. At all these wavelengths, there is unambiguous evidence for an asymmetric feature on the surface of the star, which contributes 10–15 per cent of the total observed flux. This might be due to a close companion passing in front of the stellar disc or, more likely, to large-scale convection in the stellar atmosphere.

Betelgeuse: experimental setup



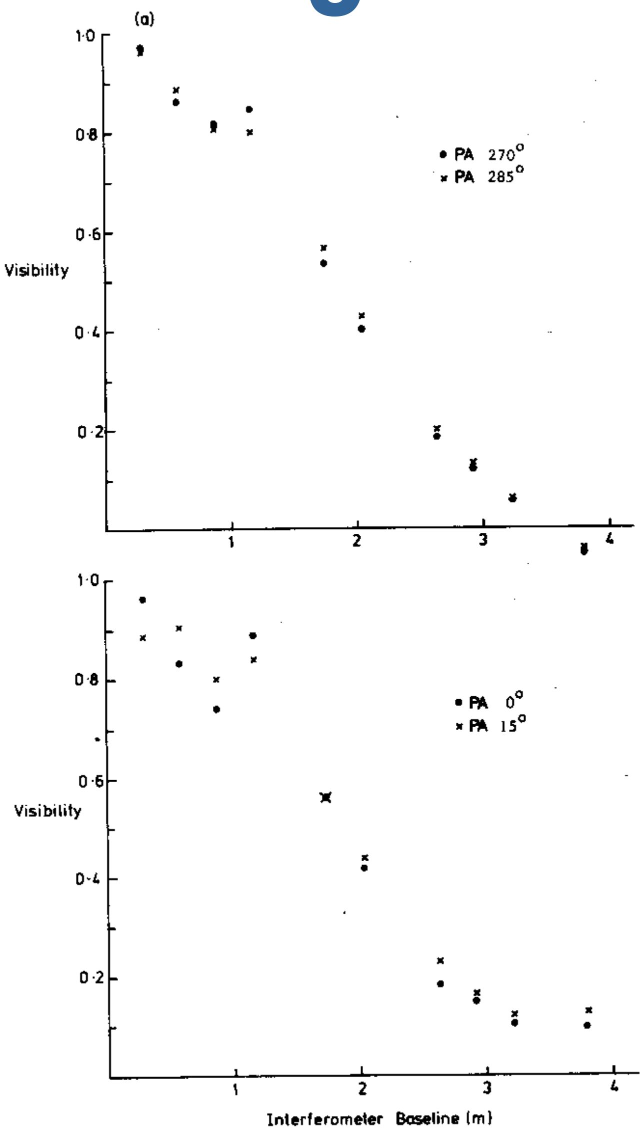




•1-d mask & 1-d CCD readout

•Rotate mask to achieve 2-d Fourier coverage.

Betelgeuse results: Fourier data



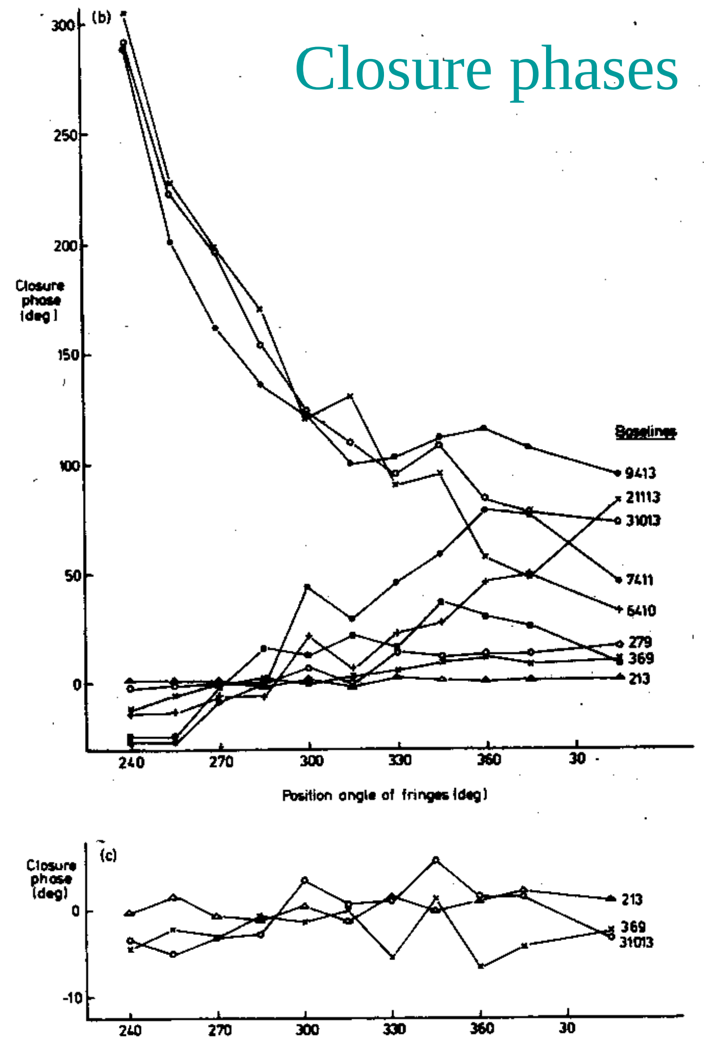


Figure 1. Calibrated visibility amplitude and closure phase data for

Betelgeuse results:interpretation

- Betelgeuse is resolved on 4m baselines.
- Betelgeuse has significant non-zero closure phases which vary slowly as a function of PA.
  - A symmetric object has all closure phases 0° or 180°.
  - Betelgeuse must be asymmetric and the asymmetry is on scales comparable with the disk size.
- Relative flux in the asymmetry must be comparable to visibility on long baselines.
  - ~10% of total flux.
- Can measure closure phase to ~degree.
  - Corresponds to relative astrometry of 3 microarcseconds with a 100m baseline.

Betelgeuse results: imaging

- Agrees with interpretation done “by hand”.
- Quantitative results from modelfit <u>after</u> image reconstruction.
- Closure phase is very important in constraining image.

Summary

- You need model-independent images.
- You need good u-v coverage.
- You need the phase, and closure phase is a good way of getting it.
- The closure phase acts as a powerful constraint in image reconstruction.